O K L A H O M A   S T A T E   U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

# ECEN 4503
# Random Signals and Noise
# Spring 2007

# Final Exam

**For all students, choose any four out of five problems.**
***Please specify which four listed below to be graded***
***:***
**1)_____; 2)_____; 3)_____; 4)_____;**

**Name : _______________________________**

**E-Mail Address:________________________________________**

**<u>Problem 1</u>**:
Assume the lifetime of a laboratory research animal is defined by a *Rayleigh* density function with $a = 0$ and $b = 30$ weeks in

$$f_X(x) = \begin{cases} \frac{2}{b}(x-a)e^{-(x-a)^2/b}, & x \geq a \\ 0, & x < a \end{cases}.$$

If for some clinical reasons it is known that the animal will live at most 20 weeks, what is the probability it will live 10 weeks or less?

**Problem 2**:
A random variable $X$ is uniformly distributed on the interval $(-\pi, \pi)$. $X$ is transformed to the new random variable $Y = T(X) = a\tan(X)$ with $a > 0$. Find the probability density function of $Y$.

**Problem 3**:
Given two random variables $X$ and $Y$, find the probability density function of the random variable $Z = X/Y$ in terms of $f_X(x)$ and $f_Y(y)$.

**Problem 4**:

Two random variables $X$ and $Y$ are related by the expression

$$Y = aX + b$$

where $a$ and $b$ are any real numbers.

(a) Show that their correlation coefficient is

$$\rho = \begin{cases} 1, & \text{if } a > 0 \text{ for any } b \\ -1, & \text{if } a < 0 \text{ for any } b \end{cases}.$$

(b) Show that their covariance is

$$C_{XY} = a\sigma_X^2 \ .$$

where $\sigma_X^2$ is the variance of $X$ .

**Problem 5**:

Two Gaussian random variables $X_1$ and $X_2$ are defined by the mean and covariance matrices

$$[\overline{X}] = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \quad [C_X] = \begin{bmatrix} 5 & -2/\sqrt{5} \\ -2/\sqrt{5} & 4 \end{bmatrix}.$$

Two new random variables $Y_1$ and $Y_2$ are formed using the transformation

$$[T] = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}.$$

Find the matrices (a) $[\overline{Y}]$ and $[C_Y]$ and (b) find the correlation coefficient of $Y_1$ and $Y_2$.